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Expression of the Radius Vector of the Moon's Orbit as disturbed by the Sun to the fifth order of small quantities.

By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

In the second volume of Delaunay's Lunar Theory the expression for $\frac{1}{r}$ is given to the fifth order of the small quantities involved. In some astronomical researches the expression for r is required, and I have thought it might be useful to other astronomers to publish the expression which I have obtained for r to the fifth order.

$$\frac{r}{a} =$$

$$(1)\ \left[1+\frac{e^2}{2}-\left(\frac{1}{6}-\frac{355}{384}e^2+\frac{e'^2}{4}-\frac{ee'^2}{2}\right)m^2+\frac{1905}{256}m^3e^2+\left(\frac{331}{288}-\frac{179}{144}e\right)m^4+\frac{83}{16}m^5\right].$$

$$(2)\ \cos l\left[-e+\frac{3}{8}e^3-\frac{5}{192}e^5-\frac{5}{2}e\gamma^4+\frac{5}{4}e^3\gamma^2+\left(\frac{11}{12}e+\frac{181}{128}e^3-\frac{19}{32}e\gamma^2+\frac{7}{8}ee'^2\right)m^2+\frac{405}{64}m^3e+\frac{8099}{256}m^4e\right].$$

$$(3)\ \cos 2l\left[-\frac{e^2}{2}+\frac{e^4}{3}+\frac{m^2e^2}{3}+\frac{1035}{128}m^3e^2\right].$$

(4) $\cos 2D\left[\left(-\frac{15}{8}e^2+\frac{45}{32}e^4+\frac{75}{16}e^2e'^2+\frac{15}{4}e^2\gamma^2\right)m-\left(1+\frac{173}{32}e^2-2\gamma^2-\frac{5}{2}e'^2\right)m^2-\left(\frac{19}{6}+\frac{32185}{1536}e^2-\frac{29}{6}\gamma^2-\frac{239}{12}e'^2\right)m^3-\frac{125}{18}m^4-\frac{709}{54}m^5\right]$.

(5) $\cos(2D-l)\left[\left(-\frac{15}{8}e+\frac{15}{16}e^3+\frac{75}{16}ee'^2+\frac{15}{4}e\gamma^2\right)m-\left(\frac{155}{32}e-\frac{423}{128}e^3-\frac{305}{32}ee'^2-\frac{145}{8}e\gamma^2\right)m^2-\frac{23689}{1536}m^3e-\frac{1011881}{18432}m^4e\right]$.

(6) $\cos l'\left[\left(\frac{3}{2}e'+\frac{27}{2}e^2e'-9e'\gamma^2\right)m^2-\frac{409}{16}m^4e'\right]$.

(7) $\cos(l-l')\left[\left(-\frac{21}{8}ee'-\frac{9}{64}e^3e'+\frac{63}{4}ee'\gamma^2\right)m-\frac{1209}{64}m^2ee'-\frac{3091}{32}m^3ee'\right]$.

(8) $\cos(l+l')\left[\left(\frac{21}{8}ee'+\frac{9}{64}e^3e'-\frac{63}{4}ee'\gamma^2\right)m+\frac{741}{64}m^2ee'+\frac{7139}{128}m^3ee'\right]$.

(9) $\cos 3l\left[-\frac{3}{8}e^3+\frac{45}{128}e^5+\frac{5}{16}m^2e^3\right]$.

(10) $\cos(2F-l)\left[\frac{5}{2}e\gamma^2+10e\gamma^4-\frac{95}{16}e^3\gamma^2-\frac{135}{16}me\gamma^2+\frac{209}{384}m^2e\gamma^2\right]$.

(11) $\cos(2D-l')\left[-\frac{35}{8}me^2e'-\left(\frac{7}{2}e'+\frac{743}{32}e^2e'-7e'\gamma^2\right)m^2-\frac{157}{8}m^3e'-\frac{3365}{48}m^4e'\right]$.

(12) $\cos(2D+l')\left[\frac{15}{8}me^2e'+\left(\frac{e'}{2}+\frac{199}{32}e^2e'-e'\gamma^2\right)m^2+\frac{91}{24}m^3e'+\frac{1025}{144}m^4e'\right]$.

(13) $\cos(2D+l)\left[-\frac{135}{64}me^3-\left(\frac{17}{16}e+\frac{1567}{256}e^3-\frac{85}{32}ee'^2-\frac{17}{8}e\gamma^2\right)m^2-\frac{151}{48}m^3e-\frac{7981}{1152}m^4e\right]$.

(14) $\cos(2D-l-l')\left[\left(-\frac{35}{8}ee'+\frac{35}{16}e^3e'+\frac{35}{4}ee'\gamma^2\right)m-\frac{1045}{64}m^2ee'-\frac{32719}{768}m^3ee'\right]$.

(15) $\cos(2D-l+l')\left[\left(\frac{15}{8}ee'-\frac{15}{16}e^3e'-\frac{15}{4}ee'\gamma^2\right)m+\frac{65}{64}m^2ee'-\frac{54613}{768}m^3ee'\right]$.

(16) $\cos D\frac{a}{a'}\left[\left(\frac{15}{16}+\frac{45}{16}e^2+\frac{15}{16}e'^2-\frac{165}{16}\gamma^2\right)m+\frac{81}{16}m^2+\frac{5497}{256}m^3\right]$.

(17) $\cos(D+l')\frac{a}{a'}\left[-\frac{5}{4}e'-\frac{25}{8}e^2e'+\frac{15}{4}e'\gamma^2+\frac{45}{8}me'-\frac{6553}{192}m^2e'\right]$.

(18) $\cos 2l'\left[\frac{9}{4}m^2e'^2\right]$. (19) $\cos(l-2l')\left[-\frac{63}{32}mee'^2-\frac{5211}{256}m^2ee'^2\right]$.

(20) $\cos(l+2l')\left[\frac{63}{32}mee'^2+\frac{1389}{256}m^2ee'^2\right]$.

(21) $\cos(2l-l')\left[-\frac{21}{8}me^2e'-\frac{1233}{64}m^2e^2e'\right]$.

(22) $\cos(2l+l')\left[\frac{21}{8}me^2e'+\frac{717}{64}m^2e^2e'\right]$.

(23) $\cos 4l\left[-\frac{e^4}{3}\right]$.

(24) $\cos 2F\left[\frac{5}{2}e^2\gamma^2-\frac{135}{16}me^2\gamma^2-2m^2\gamma^2+3m^3\gamma^2\right]$.

(25) $\cos(2D-2l')\left[-\frac{255}{32}me^2e'^2-\frac{17}{2}m^2e'^2-\frac{799}{12}m^3e'^2\right]$.

(26) $\cos(2D+l-l')\left[-\frac{315}{64}me^3e'-\frac{119}{32}m^2ee'-\frac{2879}{128}m^3ee'\right]$.

(27) $\cos(2D+l+l')\left[\frac{135}{64}me^3e'+\frac{17}{32}m^2ee'+\frac{2597}{384}m^3ee'\right]$.

(28) $\cos(2D+2l)\left[-\frac{5}{2}me^4-\frac{19}{16}m^2e^2-\frac{167}{48}m^3e^2\right]$.

(29) $\cos(2D-l-2l')\left[-\frac{255}{32}mee'^2-\frac{9835}{256}m^2ee'^2\right]$.

(30) $\cos(2D-l+2l')\left[\frac{45}{32}mee'^2+\frac{6219}{256}m^2ee'^2\right]$.

(31) $\cos(2D-2l)\left[\left(\frac{15}{8}e^2-\frac{15}{16}e^4-\frac{75}{16}e^2e'^2-\frac{15}{4}e^2\gamma^2\right)m+\frac{315}{32}m^2e^2+\frac{49209}{1536}m^3e^2\right]$.

(32) $\cos(2D-3l)\left[\frac{135}{64}me^3+\frac{6041}{768}m^2e^3\right]$.

(33) $\cos(2D-2F)\left[-\frac{75}{8}me^2\gamma^2+3m^2\gamma^2-\frac{9}{4}m^3\gamma^2\right]$.

(34) $\cos(2D-2F+l)\left[\frac{33}{16}me\gamma^2-\frac{1207}{128}m^2e\gamma^2\right]$.

(35) $\cos(2D-2F-l)\left[\frac{21}{8}me\gamma^2+\frac{15}{32}m^2e\gamma^2\right]$.

(36) $\cos 4D\left[-\frac{285}{64}m^3e^2-\frac{3}{8}m^4-\frac{1217}{480}m^5\right]$.

(37) $\cos(4D-l)\left[-\frac{2025}{512}m^2e^3-\frac{255}{128}m^3e-\frac{22887}{1536}m^4e\right]$.

(38) $\cos(4D-2l)\left[-\frac{225}{128}m^2e^2-\frac{3315}{256}m^3e^2\right]$.

(39) $\cos(D+l)\frac{a}{a'}\left[\frac{15}{16}me+6m^2e\right]$.

(40) $\cos(D+l+l')\frac{a}{a'}\left[-\frac{5}{4}ee'+\frac{45}{8}mee'\right]$.

(41) $\cos 3D\frac{a}{a'}\left[-\frac{25}{64}m^2-\frac{5}{128}m^3\right]$.

(42) $\cos(2l-2l')\left[-\frac{63}{32}me^2e'^2\right]$.

(43) $\cos(2l+2l')\left[\frac{63}{32}me^2e'^2\right]$.

(44) $\cos(3l-l')\left[-\frac{189}{64}me^3e'\right]$.

(45) $\cos(3l+l')\left[\frac{189}{64}me^3e'\right]$.

(46) $\cos 5l\left[-\frac{125}{384}e^5\right]$.

(47) $\cos(2F-l')[-3m^2e'\gamma^2]$.

(48) $\cos(2F+l')[-3m^2e'\gamma^2]$.

(49) $\cos(2F+l)\left[\frac{45}{16}e^3\gamma^2-\frac{17}{8}m^2e\gamma^2\right]$.

(50) $\cos(2F-l-l')\left[-\frac{75}{16}mee'\gamma^2\right]$.

(51) $\cos(2F-l+l')\left[\frac{75}{16}mee'\gamma^2\right]$.

(52) $\cos(2D+2l')\left[\frac{45}{32}me^2e'^2+\frac{3}{4}m^3e'^2\right]$.

(53) $\cos(2D+l-2l')\left[-\frac{289}{32}m^2ee'^2\right]$.

(54) $\cos(2D+2l-l')\left[-\frac{133}{32}m^2e^2e'\right]$.

(55) $\cos(2D+2l+l')\left[\frac{19}{32}m^2e^2e'\right]$.

(56) $\cos(2D+3l)\left[-\frac{535}{384}m^2e^3\right]$.

(57) $\cos(2D-2l-l')\left[\frac{35}{8}me^2e'+\frac{925}{32}m^2e^2e'\right]$.

(58) $\cos(2D-2l+l')\left[-\frac{15}{8}me^2e'+\frac{45}{32}m^2e^2e'\right]$.

(59) $\cos(2D-3l-l')\left[\frac{315}{64}me^3e'\right]$.

(60) $\cos(2D-3l+l')\left[-\frac{135}{64}me^3e'\right]$.

(61) $\cos(2D-4l)\left[\frac{65}{32}me^4\right]$.

(62) $\cos(2D+2F-l)\left[\frac{85}{32}m^2e\gamma^2\right]$.

(63) $\cos(2D+2F-2l)\left[\frac{75}{16}me^2\gamma^2\right]$.

(64) $\cos(2D-2F-l')\left[\frac{21}{2}m^2e'\gamma^2\right]$.

(65) $\cos(2D-2F+l')\left[-\frac{3}{2}m^2e'\gamma^2\right]$.

(66) $\cos(2D-2F+l-l')\left[\frac{77}{16}mee'\gamma^2\right]$.

(67) $\cos(2D-2F+l+l')\left[-\frac{33}{16}mee'\gamma^2\right]$.

(68) $\cos(2D-2F+2l)\left[\frac{33}{16}me^2\gamma^2\right]$.

(69) $\cos(2D-2F-l-l')\left[\frac{49}{8}mee'\gamma^2\right]$.

(70) $\cos(2D-2F-l+l')\left[-\frac{21}{8}mee'\gamma^2\right]$.

(71) $\cos(2D-2F-2l)\left[\frac{21}{8}me^2\gamma^2\right]$.

(72) $\cos(4D-l')\left[-\frac{21}{8}m^4e'\right]$.

(73) $\cos(4D+l')\left[\frac{3}{8}m^4e'\right]$.

(74) $\cos(4D+l)\left[-\frac{245}{256}m^4e\right]$.

(75) $\cos(4D-l-l')\left[-\frac{2975}{256}m^3ee'\right]$. (76) $\cos(4D-l+l')\left[\frac{765}{256}m^3ee'\right]$.

(77) $\cos(4D-2l-l')\left[-\frac{525}{64}m^2e^2e'\right]$. (78) $\cos(4D-2l+l')\left[\frac{225}{64}m^2e^2e'\right]$.

(79) $\cos(4D-3l)\left[-\frac{225}{512}m^2e^3\right]$. (80) $\cos(4D-2F)\left[-\frac{3}{2}m^2\gamma^2\right]$.

(81) $\cos(4D-2F-l)\left[-\frac{45}{32}m^2e\gamma^2\right]$. (82) $\cos(D-l')\frac{a}{a'}\left[-\frac{15}{16}me'+\frac{1057}{64}m^2e'\right]$.

(83) $\cos(D-2l')\frac{a}{a'}\left[-\frac{435}{128}me'^2\right]$. (84) $\cos(D+2l')\frac{a}{a'}\left[\frac{255}{128}me'^2\right]$.

(85) $\cos(D+l-l')\frac{a}{a'}\left[-\frac{15}{8}mee'\right]$. (86) $\cos(D+2l)\frac{a}{a'}\left[\frac{135}{128}me^2\right]$.

(87) $\cos(D+2l+l')\frac{a}{a'}\left[-\frac{45}{32}e^2e'\right]$.

(88) $\cos(D-l)\frac{a}{a'}\left[-\frac{15}{16}me-\frac{1233}{128}m^2e\right]$.

(89) $\cos(D-2l)\frac{a}{a'}\left[-\frac{465}{128}me^2\right]$. (90) $\cos(D-2l+l')\frac{a}{a'}\left[\frac{115}{32}e^2e'\right]$.

(91) $\cos(D-2F)\frac{a}{a'}\left[-\frac{45}{8}m\gamma^2\right]$. (92) $\cos(D-2F+l')\frac{a}{a'}\left[\frac{5}{3}e'\gamma^2\right]$.

(93) $\cos(3D-l')\frac{a}{a'}\left[-\frac{125}{64}m^2e'\right]$. (94) $\cos(3D+l')\frac{a}{a'}\left[-\frac{15}{16}m^2e'\right]$.

(95) $\cos(3D+l)\frac{a}{a'}\left[-\frac{35}{64}m^2e\right]$. (96) $\cos(3D-l)\frac{a}{a'}\left[\frac{775}{128}m^2e\right]$.

(97) $\cos(3D-l+l')\frac{a}{a'}\left[-\frac{75}{32}mee'\right]$. (98) $\cos(3D-2l)\frac{a}{a'}\left[\frac{175}{64}me^2\right]$.

(99) $\cos(3D-2F)\frac{a}{a'}\left[\frac{25}{16}m\gamma^2\right]$. (100) $\cos(2F-2l)\left[-\frac{5}{2}e^2\gamma^2+\frac{135}{16}me^2\gamma^2\right]$

(101) $\cos(2D-2l-2l')\left[\frac{255}{32}me^2e'^2\right]$.

(102) $\cos(2D-2l+2l')\left[-\frac{45}{32}me^2e'^2\right]$.

(103) $\cos(D-l+l')\frac{a}{a'}\left[\frac{5}{4}ee'-\frac{45}{8}mee'\right]$.

(104) $\cos(D-l-l')\frac{a}{a'}\left[\frac{75}{32}mee'\right]$.

Radcliffe Observatory, Oxford:
1893 *June* 9.

Occultation of Saturn, 1893 *May* 25, *observed at Sydney Observatory.*

(*Communicated by H. C. Russell, B.A., F.R.S., Government Astronomer.*)

Atmosphere was fine and clear, definition good. The definition of the Moon's limb as it passed over the planet at ingress was remarkably sharp as I saw it with a power of 400 on the 11½-inch Equatoreal. Mr. Sellors observed with a 6-inch Grubb telescope, and Mr. Lenehan with a 3-inch (private) telescope. Time used, Sydney mean time.

Ingress.	H.C.B.	H.A.L.	R.P.S.
	h m s	h m s	h m s
First contact ring	5 19 3·4	...	...
„ „ limb	...	5 19 22·0	5 19 16·4
Planet bisected	5 19 36·4	...	...
Last contact limb	5 19 58·4	5 20 7·0	5 19 58·2
„ „ ring	...	...	5 20 13·1

At egress Mr. Russell attended to photographs, Mr. Sellors observed with the 7½-inch guiding telescope of the star camera, Mr. Lenehan with the 3-inch.

Egress.	H.A.L.	R.P.S.	
	h m s	h m s	
First contact ring ...	6 39 10	6 39 5	
„ „ limb ...	...	...	
Last contact limb ...	...	6 39 50	uncertain to the extent of 4 or 5 seconds.
„ „ ring ...	6 40 4	6 40 15	

Observatory, Sydney:
1893 *May* 26.

Results of Micrometer Comparisons of Saturn and γ^1 *Virginis in* 1893 *April.* By John Tebbutt.

Filar-micrometer comparisons were obtained of *Saturn* and the preceding or north component of γ *Virginis* with the 8-inch equatoreal on the evenings of April 3, 4, 5, 6, 8, 10, 11. On April 3, 4, 5, 10, 11, ten comparisons of the planet's preceding and north, and the same number of the following and south limbs were observed. On the 6th ten comparisons of the preceding and north limbs were secured, but, owing to clouds, the number of comparisons of the following and south limbs was limited to seven. On the 8th ten comparisons of the following and north, and the same number of the following and south limbs were made. On this date the preceding limb and the star crossed